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Market Crashes and Informational Avalanches

IN HO LEE
University of Southampton

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This paper analyses a security market with transaction costs and a sequential trading structure. Transaction costs may prevent many traders from revealing their private information if they trade in a sequential fashion. Due to the information aggregation failure, hidden information gets accumulated in the market which may be revealed by a small trigger, yielding a high volatility in the absence of an accompanying event. The paper first characterizes the optimal trading strategy of the agent which constitute the unique equilibrium. Further properties of the price sequence are obtained using the concepts of informational cascade and informational avalanche.

The results are applied to the explanation of market crashes. In particular, the dynamics of market crashes are illustrated as evolving through the following four phases: (1) boom; (2) euphoria; (3) trigger; and (4) panic; where the euphoria corresponds to the informational cascade and the panic corresponds to the informational avalanche.

Bull markets are built on the walls of worry.—Anonymous

1. INTRODUCTION

Recent empirical research in finance reveals a variety of discrepancies of financial data from the standard theory drawing from rational expectations. For instance, high frequency data from major financial markets exhibit a greater volatility than can be explained by informational events around the movements. To explain these observations, papers relying on market micro-structure theory focus on the trading arrangements in the financial markets. The present paper follows this stream of research to construct a model which explains volatile price movements in the absence of accompanying news.

We consider a financial market where agents have to pay transaction costs to trade and they trade sequentially. We establish that such a market could be highly volatile if it fails to aggregate information given to the agents. The failure in information aggregation takes place because traders place a disproportionate weight on the previous price history in an attempt to utilize public information in their investment decision making. For instance, observing many trading orders based on good news, a trader with bad news tends to discount his own bad news and makes an investment choice close to the one based on good news. When mildly bad news is combined with the transaction cost, the choice based on bad news may become indistinguishable from the one based on good news. It follows that the price may remain high even if there are many traders with bad news.

The presence of many hidden-news traders poses a potential danger of a sudden surge in market volatility, if they all come to realize the possibility of an alternative state other than implied by the present price. It may not take a big informational event to the economy to get them suspicious about their previous investment decision. If they all had hidden private information inconsistent with the present price, a single trigger that implies the other state strongly suffices to make them suspicious. After the trigger, a surge in market volatility follows only if there have been many traders with private information inconsistent with the present price.

The mechanism of information aggregation failure relies on the notion of informational cascades (Bikhchandani, Hirshleifer and Welch (1992) and also herd behaviour Banerjee (1992)). An informational cascade is an event in which a sequence of agents takes identical actions as they try to exploit the information available from the history of previous action choices. The sudden reversal of informational cascades is explained using the concept of informational avalanches. Informational avalanches exploit the property that non-fully revealing informational cascades are fragile.

The model posits a setting in which information is dispersed throughout the economy in the form of private signals. Each agent makes an investment decision based on his own private information and the history of previous agents' decision. Assuming that the price of the security is set by a market maker equal to the expected fundamental value of the security conditional on the history of trade orders, the public belief conditional on the history is reflected in the price.

We first solve for the optimal trading strategy of the agent which constitutes the unique equilibrium of the model. The agent trades at most twice, first to exploit the informational advantage due to the private signal and second to unload the risky asset holding at the price which reflects the private signal revealed through the first trade. Further properties of the equilibrium are obtained including the implication of the informational avalanche on the volatility and informativeness of the equilibrium price and long-run distribution of the price.

Later we apply the results of the analysis to explain the phenomena of market crashes. The equilibrium of the market considered in the model exhibits a high volatility when the informational cascade is reversed by a trigger. If there is a sudden change in public beliefs, then the price may change drastically which is described as a market crash. The model explains the dynamics of market crashes through four phases: (1) boom; (2) euphoria; (3) trigger; and (4) panic; where euphoria corresponds to the informational cascade and panic corresponds to the informational avalanche. Hence a market crash is described as a procedure which corrects a public belief which is inconsistent with the current distribution of private information.

There have been a few theoretical attempts to construct a model which can generate big price movements without substantial news.[1] Caplin and Leahy (1991) construct a model in which many investors change their investment choice suddenly. The sudden change in their model follows from a revelation of new information that becomes available only when some investors collect significant private information. Therefore the economy before the crash is not regarded as a state in which information aggregation fails but it is in the state of waiting for the accumulation of information. In contrast, in our model the stock market before the crash is in a boom although the true private information would reveal the opposite. Zeira (1993) explains the phenomenon of price overshooting and crashes through a learning process. He considers a learning model in which agents do not know when the increase in the fundamental stops. He suggests that the entry of new investors may have caused the two crashes of the century. Romer (1993) considers two models to explain price movements without accompanying news. His second model is

1. There were other papers which focused on explaining the '87 crash. Among them, we discuss about Genotte and Leland (1990) and Jacklin, Kleidon and Pfeiderer (1992) in Section 5 when we investigate the empirical validity of our model.

particularly similar to the present one in that it characterizes a security market with dispersed information and transaction costs. He assumes that immediate processing of private information for trade is more costly than delayed processing. Due to the incentive to save information processing costs, the time of trade is uncoupled from the time of information acquisition and the price may move long after agents acquire information. However, his model does not provide a mechanism through which small information accumulates without being revealed through trading since delay in the trade occurs uniformly. In contrast, agents in our model delay their trades only when there have been a sequence of trades which have revealed substantial information and trade only after a trigger so that the model is able to explain the process of price build-up and subsequent burst. Most importantly the present model relies on a mechanism which highlights the possibility of learning from price movements. Bulow and Klemperer (1994) constructed a model of frenzies and crashes based on strategic interactions of rational agents in an auction setting. The present paper provides an explanation complementary to these attempts using a model in which agents learn from other's action choice.

There are explanations of market crashes which rely on the irrationality of agent's behaviour. Shiller claimed that the financial markets do not conform to the efficient market hypothesis due to the irrational behaviour of investors. After the crash of 1987, he conducted a survey of investors active around the crash (Shiller (1988)). On the basis of the results he argued that the crash took place because of a sudden change in the investors' investment pattern. French (1988) provided an explanation similar to this paper. He did not formalize the intuition in a theoretical framework and it also appears that he thought the explanation requires the irrationality of investors. This paper formally shows that the market crash may happen due to a failure in information aggregation even if agents are rational.

Recently there has been a lot of interest in the herd behaviour model among economists. Gul and Lundholm (1995) address the issue of timing decision in a sequential decision model which is one of the major issues of the present paper. As was noted in the survey by Devenow and Welch (1995), however, there have not been many attempts to apply the framework to asset pricing. Avery and Zemsky (1995) analyse the problem of asset pricing in a financial market with sequential trading structure.

The rest of the paper is organized as follows. Section 2 formalizes the intuition into a model with a sequential trade structure and transaction cost. Section 3 introduces key concepts in information aggregation which help characterize the dynamics of market crashes. Section 4 analyses the evolution of the security market with transaction cost and establishes its properties. Section 5 explains the market crash in the framework of the model analysed in Section 4. Also important features of our model are discussed and related to historical events. Section 6 concludes.

2. SECURITY MARKET MODEL WITH TRANSACTION COSTS

The model has two assets, cash and a risky asset whose ex post liquidation value Y depends on the state of nature. A sequence of risk averse agents maximize the expected utility from holding assets conditional on the information available at the time of decision. The utility depends on the final wealth and the transactions made over the entire trading rounds: $u(W, z)$ where W is the final wealth and z is the vector of trading orders made over the entire trading rounds. Each agent i is endowed with the same initial wealth but different private signal θ^i correlated with the liquidation value of the risky asset. They trade the risky asset against a single market maker over trading rounds, $t = 1, \dots, T+1$, where T is the total number of agents in the market.[2] At the end of each trading round, the liquidation value becomes known with probability β while at the end of the trading round $T+1$ the liquidation value of the risky asset will be known with certainty.[3] In each trading round a new agent arrives in the market while the agents who have arrived in the previous trading rounds remain active in the sense that they may readjust their position if need arises. The order of agents, and equivalently the order of the private signals arriving in the market, is taken as a part of the stochastic environment in the model. If the agents decide to make a non-zero trading order against the market maker, they incur a transaction cost c only once for the first non-zero trade. We assume that the transaction cost is independent of the order size and also separable from the utility of pecuniary wealth.[4]

Given description, the agent i's optimization problem at trading round t is written as[5]

$$\max E_t^i\{u(W_t^i + Yx_t^i + \textstyle\sum_{\tau=t}^{T+1}(Y-p_\tau)z_\tau^i, \{z_\tau^i\}_{\tau=t}^{T+1})\}, \tag{1}$$

where E_t^i is the expectation operator conditional on agent i's information at trading round t, p_t is the risky asset price at trading round t, $\{z_\tau^i\}_{\tau=t}^{T+1}$ is the agent i's trade orders at trading rounds t to $T+1$, W_t^i is agent i's cash holding at the beginning of trading round t, and x_t^i is agent i's risky asset holding at the beginning of trading round t.

It is worth noting that agents' information sets include the whole price history up to the present. We denote the price history in trading round t by p^t: $p^t = \{p_0, p_1, \dots, p_t\}$.

The price of the risky asset is posted by the market maker who will buy and sell against any trade order placed at the posted price.[6] In determining the price for each trading round, the market maker uses all information available from the history of trade orders placed in the previous rounds up to the current one.

We make the following assumptions to simplify the analysis.

Assumption 1. Agents behave as price takers.

Assumption 2. Agents have 0 initial cash holding and 0 initial risky asset holding: $W_t^t = 0$ and $x_t^t = 0$.

Assumption 3. Agents have the following utility function:

$$u(W, z) = -\exp[-W] - c \cdot \mathscr{I}_{\{z \neq 0\}},$$

where W is the pecuniary wealth and z is the trading order and $\mathscr{I}_{\{\cdot\}}$ is the indicator function.

2. To enable trader T to readjust his portfolio as all other traders are allowed to, the market opens $T+1$ rounds.

3. The probability β is assumed small but strictly positive so that there is uncertainty each period. In particular it prevents an agent from making an infinite trading order for a certain trading profit in any period.

4. The assumption of separability of the transaction cost is mostly innocuous. Most of equilibrium characterization remains the same without the separability since the exponential utility function allows no income effect. However, non-separable transaction cost imposes computational burden in the intermediate steps. Moreover, we imagine that the transaction cost represents not only the order processing cost but also other costs in trading such as time cost in engaging in the trading process and the order processing cost occupies a negligible part in the final pecuniary wealth.

5. Since agent i first participates in the trading in round $t = i$, the optimization problem for agent i is defined only for $t \geqq i$.

6. This simplifying feature of our model is not crucial in deriving the result. Alternatively one can work with changing inventory of security and cash after each trading round imposing *ad hoc* allocation rules when the order cannot be filled with the inventory.

Assumption 4. There are two states of nature, G and B, $Y = \{G, B\}$. The liquidation value of the asset is 1 in state G and 0 in state B.

Assumption 5. The initial prior of the states is non-degenerate: $\mu_0 \notin \{0, 1\}$ where μ_0 is the probability of the state G before trading round 1.

Assumption 6. There are N (finite) private signals which satisfy the monotone likelihood ratio property

$$0 < \lambda_1 < \cdots < 1 < \cdots < \lambda_N < \infty$$

where $\lambda_n = q_{nB}/q_{nG}$ and q_{nY} denotes the probability of getting the signal $\theta_n \in \Theta = \{\theta_1, \ldots, \theta_N\}$ conditional on the state Y.

Assumption 1 simplifies the analysis by ruling out strategic behaviour of the agents. First the agents are not allowed the strategy which misrepresents the private signal, namely placing a trading order which is optimal for a private signal the agents do not have. Second when making the decision whether to place a non-zero trading order in each period, the agents do not consider the consequence on the price path from not trading.[7] Assumption 6 implies that signals are informative and their information contents are distinct.

Consistent with the notation in Assumption 5, we denote the public belief about the state G at trading round t as μ_t. We denote agent i's private belief at trading round t by π_t^i, which may vary for different i due to the private signal.

Next we define the equilibrium in the risky asset market in the fashion of Bayesian Nash equilibrium.

Definition 1. The equilibrium in the risky asset market consists of a price sequence and a trading strategy sequence for each agent, $(p_t, \{z_t^i\}_{i=1,\ldots,t})$ such that

1. $p_t = E[Y|\mu_0, z_1, \ldots, z_{t-1}]$, where $z_t = \{z_t^i\}_{i=1,\ldots,t}$,
2. $z_t^i \in \arg\max E_t^i\{u(W_t^i + Yx_t^i + \sum_{\tau=t}^{T+1}(Y - p_\tau)z_\tau^i, \{z_\tau^i\}_{\tau=t}^{T+1})\}$, for all i for all t.

The first equilibrium condition implies that the market maker determines the price such that it is equal to the expected value of the risky asset at liquidation.[8] The second condition means that agents maximize their profit at each moment based on the information available then. Together they should be consistent in the sense that the price sequence correctly reflects the decision procedure of the agent and agent's optimal decision is consistent with information that can be inferred from the price sequence about the private signals.

7. To understand these two restrictions, suppose that each period there arrive at least three agents with the same signal so that the true signal is revealed by the identical non-zero tradings placed by two agents. It is straightforward to see that the trading strategy satisfying the two restrictions constitutes a Nash equilibrium.

8. The feature that the market maker is not maximizing the profit can be relaxed without changing the main result. As Smith and Sorensen (1996) show, the informational property of this class of models is robust to the introduction of noise. Therefore it would be possible to construct a model where the market maker may recoup the loss due to the informed traders by trading against noise traders. See also Kyle (1985) for a similar approach and a justification.

3. INFORMATIONAL CASCADES AND INFORMATIONAL AVALANCHES

The sequential trading structure combined with the transaction cost sometimes prevent the trading orders from revealing the private signals underlying the orders. In particular, a sequence of agents with bad news may behave the same as those with good news. Conversely a single trading order which reveals a surprise can induce all the traders who have previously behaved identically to distinguish themselves by placing different orders. It there are many agents whose information was not reflected in the market price due to indistinguishable orders in the previous trading rounds, the simultaneous aggregation of those signals may bring about a big change in the risky asset market. We introduce two concepts illustrating these two phenomena.

Definition 2. An informational cascade with respect to $\theta \in \hat{\Theta}(\mu) \subseteq \Theta$ develops if given the public belief μ, $z_t^t(\theta) = 0$ for all $\theta \in \Theta$ with Θ having more than 2 elements.

The informational cascade is an event in which traders with different private signals may not reveal themselves because they all place 0 trading orders when they first arrive in the market. The definition of the informational cascade differs from that of Bikhchandani *et al*. (1992) in that it allows a partial informational cascade because Θ can be a proper subset of Θ. To allow for an occurrence of a subsequent informational avalanche, it is necessary that some signals are excluded from the set of signals indistinguishable in terms of the trade orders they induce. We call the set Θ the *informational cascade signal set* in the following.

Definition 3. An informational avalanche occurs in trading round t if $z_t^i \neq 0$, $i < t$, for agent satisfying $z_s^i = 0$ for $s = i, \ldots, t-1$.

An informational avalanche takes place if some agents who have arrived in the previous trading rounds but have not made a non-zero trading order yet make a non-zero trading order. As will be shown, non-zero trading orders reveal their private signal completely, and thus the occurrence of the informational avalanche is a procedure in which hidden information during the informational cascade gets revealed to market.

To understand our explanation of market crashes, it is important to understand how informational avalanches reverse the momentum gained in informational cascades. When a partial informational cascade develops, the market may accumulate hidden information not consistent with the prevailing price. Informational avalanches take advantage of the fragility of the informational cascade combined with the trigger made by the signal not included in the informational cascades. In particular, the signal not included in the informational cascade may imply a state different from the one consistent with the current informational cascade. When an action induced by such a signal is observed in the market, the public belief gets diffused in the sense that the probability weight given to the state implied by the informational cascade gets smaller. Given the new diffused public belief, signals which were not distinguishable in terms of non-zero trading orders during the informational cascades may now induce distinguishable optimal orders so that previously hidden private signals can be revealed to the public.

4. SECURITY MARKET EQUILIBRIUM

This section establishes a few key properties of the informational cascade and the informational avalanche with which we explain the market crash in the subsequent section.

We start with a characterization of the equilibrium in terms of an agent's optimal trading strategy. The optimal trading strategy induces a stochastic process for the security price. The analysis of the stochastic process reveals the properties of the informational cascade and the informational avalanche.

4.1. *Optimal trading strategy*

Since agents are allowed to trade more than once, they solve a dynamic programming problem with a rational expectation on price evolution as well as the liquidation value of the risky asset. The main theorem establishes that each agent optimally trades at most twice if the gain from trading exceeds the cost of transaction. The first trade enables the agent to exploit the informational advantage due to the private signal while the second trade allows the agent to adjust a risky position once the price is aligned with the private belief. We derive the main result throught a few steps.

Taking account of the fact that the liquidation value of the asset is known with probability β at the end of each period, the agent's optimization problem in (1) is reformulated recursively.

For $t = T+1$,

$$V_{T+1}(\pi^i_{T+1}, p_{T+1}, W^i_{T+1}, x^i_{T+1})$$
$$= \max E_{T+1}\{-\exp[-(W^i_{T+1} + Y(x^i_{T+1} + z^i_{T+1}) - p_{T+1} z^i_{T+1})]\} - c \cdot \mathscr{I}_{\{z^i_{T+1} \neq 0\}}. \qquad (2)$$

For $t \leqq T$,

$$V_t(\pi^i_t, p_t, W^i_t, x^i_t) = \max \beta E_t\{-\exp[-(W^i_t + Y(x^i_t + z^i_t) - p_t z^i_t)]\} - c \cdot \mathscr{I}_{\{z^i_t \neq 0\}}$$
$$+ (1-\beta) E_t V_{t+1}(\pi^i_{t+1}, p_{t+1}, W^i_{t+1}, x^i_{t+1}), \qquad (3)$$

where $p_{t+1} = \Pr/(G|\mu_t, z_t)$, $W^i_{t+1} = W^i_t - p_t z^i_t$, and $x^i_{t+1} = x^i_t + z^i_t$.

The recursive formulation of the optimization problem allows the characterization of the optimal trading strategy as the solution to a simpler problem in the following theorem.

Theorem 1. 1. *The agent trades at most twice, first to buy or sell the risky asset based on the private signal and second to unload the risky asset holding at the fair price after the price reflects the private signal due to the first trading.*

2. *The first non-zero optimal trading order is the solution to the following optimization problem while the second non-zero optimal trading order is the negative of the first optimal trading order*:

(*i*) *for* $t = T+1$,

$$\Phi_{T+1}(\pi^i_{T+1}, p_{T+1}) = \max_z E_{T+1}\{-\exp[-(Y - p_{T+1}) z^i_{T+1}]\} - c;$$

(*ii*) *for* $t \leqq T$,

$$\Phi_t(\pi^i_t, p_t) = \max_z \beta E_t\{-\exp[-(Y - p_t) z^i_t] + (1-\beta) E_t\{-\exp[-(p_{t+1} - p_t) z^i_t]\} - c.$$

3. *The agent makes the first non-zero trading order in period t if and only if*

$$\Phi_t(\pi^i_t, p_t) \geqq -1.$$

It is useful to understand the meaning of terms in the optimization problem in part 2(ii) of the theorem. The risky asset holding if taken in the current trading round pays off either through the second trading in the reverse direction or when the liquidation value is known which takes place with a certain probability at the end of each trading round. The last term c is the transaction cost due to a non-zero trading in the current period. The first term represents the expected payoff if the liquidation value is known at the end of trading in round t before the agent unloads it through trading while the second term represents the expected payoff if the agent unloads it in trading round $t+1$. The agent makes a non-zero trade as in part 2 if and only if the expected payoff from such a strategy is bigger than the best possible expected payoff from trading in the future. Due to the price taking behaviour in Assumption 1, the agent compares the profit from optimal non-zero trading against the payoff from no trading permanently and thus part 3.

The theorem is proved through two intermediate lemmas. Consistent with the dynamic programming approach, the first lemma characterizes optimal trading after the price reflects the private signal through a non-zero trading order. The second lemma establishes that the first non-zero trading order of the agent fully reveals the private signal so that the condition of the first lemma is met in the second trading. Together the lemmas explain how to reformulate the original problem to obtain the one in the theorem.

Lemma 1. *Suppose $\mu_t = \pi^i_t$. If there is no further transaction cost for trading, then $z^i_t = -x^i_t$.*

Proof. See Appendix. ||

Corollary 1. *Suppose $\mu_t = \pi^i_t$ and $x^i_t = 0$. Then $z^i_t = 0$.*

Proof. See Appendix. ||

Lemma 1 implies that when the private belief is the same as the public belief, the agent unloads any risky asset holding unless the transaction cost exceeds the trading gain, in which case the agent optimally chooses not to trade. Corollary 1 implies that the agent does not trade once the agent has neither informational advantage nor a risky asset holding. Since the agent solves the optimization problem with a rational expectation as to the future evolution of the market, the initial optimal trading order should take account of the result in Lemma 1 and Corollary 1 if the first non-zero trading order indeed fully reveals the private signal underlying the order. The next lemma establishes that the non-zero trading order fully reveals the private signal.

Lemma 2. *Consider $\theta, \theta' \notin \hat{\Theta}_t$, such that $\lambda_\theta \neq \lambda_{\theta'}$ where $\hat{\Theta}_t$ is the informational cascade set in trading round t. Then $z^i_t(\theta) \neq z^i_t(\theta')$ where $z^i_t(\theta)$ is the optimal trading order of agent i with the private signal θ.*

Proof. See Appendix. ||

Using the results of Lemmas 1, 2 and Corollary 1, we are prepared to prove Theorem 1.

Proof of Theorem 1. Lemma 2 implies that the first non-zero trade fully reveals the private signal while Lemma 1 implies that if there is no informational advantage the agent trades only to unload the risky asset holding taken in the first non-zero trade. Corollary

1 indicates that there is no further trading after the two non-zero trades. Hence the optimal trading strategy in the first part of the theorem follows.

To prove the second part consider the recursive formulation of the optimization problem in equations (2) and (3). The objective function for trading in round $T+1$ is obvious since there is no further opportunity for trading. For $t \leqq T$, the result in Part 1 implies that equation (3) can be written as

$$V_t(\pi^i_t, p_t, 0, 0) = \max - c \cdot \mathscr{I}_{\{z^i_t \neq 0\}} + \beta E_t\{-\exp[-(Y-p_t)z^i_t)]\} + (1-\beta)E_t\{-\exp[-(p_{t+1}-p_t)z^i_t)]\}.$$

Part 3 follows from the observation that the agent makes a non-zero trade if and only if waiting for the future does not improve the expected payoff. However the price-taking behaviour restricts the agent to not compare the profit from the optimal non-zero trade against the future trading profit from the price path which does not reflect the private signal. Therefore the future expected profit would be -1. ||

4.2. *Properties of equilibrium price path*

In the remainder we consider the evolution of the market as the number of trading rounds tends to infinity so that the value function needs no reference to t. The sequence of the security price constitutes a stochastic process driven by the stochastic process of the private signals generated conditional on the underlying state of nature. We explore the property of the stochastic process and the optimal trading behaviour consistent with the stochastic process. All remaining proofs are provided in Appendix.

Proposition 1. $E[p_{t+1}|p_t] = p_t$.

The sequence of the security price exhibits the martingale property and thus the current price is the best predictor of the future price. However, the martingale property of the price sequence should be understood with care. In particular, it does not rule out a dramatic price movement over a short period of time.

The optimal trading strategy may prevent the agent making a non-zero trading order if the gain is outweighed by the cost. The observation leads to the following proposition on the development of the informational cascade.

Proposition 2. *There exist unique $\bar{\mu}(\theta)$ and $\underline{\mu}(\theta)$ such that for $\bar{\mu}(\theta) \geqq \mu \geqq \underline{\mu}(\theta)$ the agent newly arriving in trading round t with a private signal θ places a non-zero trading order.*[9]

The proposition establishes that the informational cascade develops as the belief gets concentrated either on state G or B. Hence private signals with moderate informational content may not get revealed during the informational cascade.

The evolution of the security price is further investigated in the next two propositions. The sequential trading structure under the transaction cost may prevent the correct information aggregation even in the long run which follows from the eventual development of total informational cascades. On the other hand, the turbulence in the asset market during the informational avalanches results from the price correction to reflect the information distributed in the economy at the moment.

9. It is possible that the transaction cost is big relative to the information due to the private signal that there is no μ for which the expected profit is positive, in which case we set $\bar{\mu}(\theta) = \underline{\mu}(\theta) = \frac{1}{2}$.

Proposition 3. *Suppose that in trading round t, $z_t^i \neq 0$ for some $i < t$. Then*:

1. $\mathrm{Var}[p_{t+1}] \geqq \mathrm{Var}[E[Y|p_t, z_t^t]]$;
2. $E[(p_{t+1} - Y)^2 \leqq E[(E[Y|p_t, z_t^t] - Y)^2]$ *where Y is the true liquidation value*.

The first part of Proposition 3 claims that the variability of the price at the time of the informational avalanche exceeds what would be possible if only the trading order from the new agent is taken into account. Therefore the price may move more violently under the informational avalanche than otherwise. The excessive variability comes from the revelation of the signals which have been hidden in the informational cascade, but suddenly and simultaneously revealed in the informational avalanche. It implies that a dramatic price movement may happen even in the absence of correspondingly dramatic news at the time of price change. Later we explain market crashes using this intuition.

The second part of the proposition implies that when the informational avalanche occurs, the subsequent price is likely to be closer to the correct one. Although we cannot entirely rule out the possibility of the price being incorrect after the informational avalanche, the precision represented by the mean squared error of the price is smaller than the one which only takes account of the signal newly arriving in the market. Because the informational avalanche incorporates the hidden information accumulated during the informational cascade into the price and thus no further surprise can take place, the market will stay at the new price level established through the informational avalanche.

Next we investigate the long-run property of the security price sequence.

Proposition 4. 1. $\Pr\{\lim_{t\to\infty} z_t^t(\theta) = 0, \text{ for all } \theta \in \Theta\} = 1$.
2. $\Pr\{\lim_{t\to\infty} \arg\min_Y |Y - p_t| \neq Y\} > 0$ *where Y is the true liquidation value*.

The first part of Proposition 4 implies that a total informational cascade where no signals induce a non-zero trading order will develop eventually. When the total informational cascade develops, no private signal can be distinguished based on the trading order since it is identically zero conditional on all signals. While the property implies that the price movement will stabilize in the long-run, it also indicates that the market may fail to aggregate information correctly since new information arriving after the development of the total informational avalanche will not be reflected in the security price.

Indeed the second part of the proposition shows that the correction procedure implied by the informational avalanche is imperfect since there is a strictly positive probability that the market price does not reflect the whole information available in the market even in the long run. The property underscores the fact that the present model may have quite different predictions as to the information aggregation capability of the security market from the standard rational expectations model.

5. A SIMPLE MODEL OF MARKET CRASHES

Most market crashes are characterized by the following three stylized facts.

1. At the time of the market crash, no major event changing the state of nature happens.
2. Before the market crash, the price rises steadily for a substantial length of time.
3. After the market crash, the price remains low for a substantial length of time.

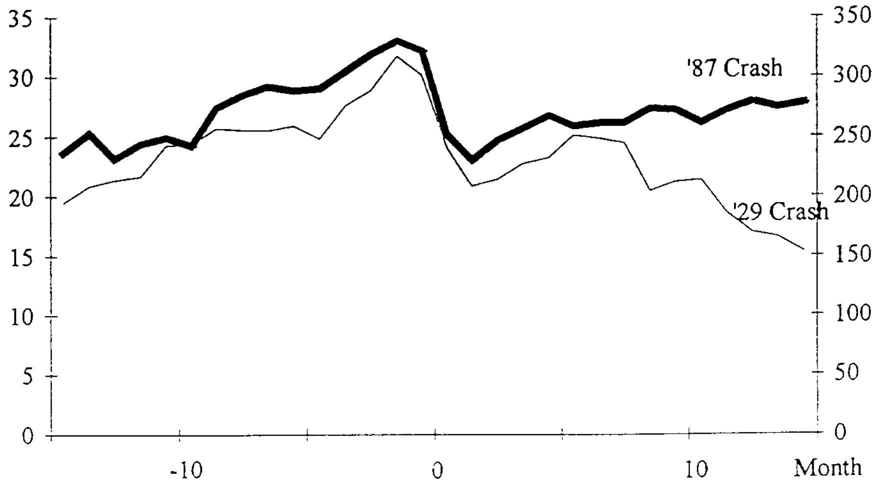

FIGURE 1

Stock price movement (S&P 500). (1) '29 Crash series covers the monthly price movements for 30 months around the Crash. (2) '87 Crash series covers the monthly price movements for 30 months around the Crash. (3) '29 Crash series contains 90 firms and '87 Crash series contains 500 firms. Data Source: CRSP.

As can be seen in Figure 1, the two market crashes in this century exhibit the same pattern of price movement illustrated by the stylized facts (2) and (3). The price has risen for many years before the crash and remained low at least a year before rising again. Also economists have not been able to identify any catastrophic event around the market crashes except the crashes themselves.

The first stylized fact is an essential feature of market crashes; without this feature there is nothing intriguing about these events because prices should change a great deal with a big change in the state of nature such as a war or a bad harvest. Under the presumption that the first stylized fact is valid, the second one and the third one contradict the efficient market hypothesis. Given enough time to reveal information, the market should have learned the correct underlying parameter of the security price so that such a big price fall should not occur and be subsequently sustained for a long time. On the other hand if the price drop is triggered by a trading strategy independent of the state of nature, there is no reason why prices should not revert to a high level before long.

The previous section established a few properties of the price process in a sequential security market with transaction costs. This section applies the findings to explain the price evolution characterized by the stylized facts of market crashes.

5.1. *Characterization of price path over market crashes*

To explain the price path consistent with the stylized facts of market crashes, we further simplify the model and we assume that there are three private signals, H, L, and R.[10] We consider a particular sequence of these private signals and show that such private signal sequence generates a price path consistent with the stylized facts.

10. The name of each signal has the connotation of High (H) and Low (L) liquidation values while R corresponds to Rare meaning that the signal is observed with only a small probability.

The three private signals have the following information matrix:

$$\begin{pmatrix} q_{HG} & q_{HB} \\ q_{LG} & q_{LB} \\ q_{RG} & q_{RB} \end{pmatrix}$$

where $q_{\theta s}$ denotes the probability of signal θ conditional on state s.

The signals provide information as to the underlying state of nature and have likelihood ratios satisfying $\lambda_H < 1 < \lambda_L < \lambda_R$ where $\lambda_\theta = q_{\theta B}/q_{\theta G}$. In addition they satisfy the following conditions: $\bar{\mu}(R) > \bar{\mu}(H) > \bar{\mu}(L) > \underline{\mu}(L) > \underline{\mu}(H) > \underline{\mu}(R)$ where $\bar{\mu}(\theta)$ and $\underline{\mu}(\theta)$ are as defined in Proposition 2. We also assume that signals H and L are frequently observed while the signal R is only rarely observed under both states, namely q_{RG} and q_{RB} are small relative to other probabilities in the information matix.

Next consider a signal sequence which contains H in the first part and L in the next part followed by R: $(H, H, H, \ldots, L, L, \ldots, R)$. The arrival of private signals in such a sequence is characterized by four phases: (1) boom; (2) euphoria; (3) trigger; and (4) panic.

Phase 1. Boom: Given the public belief μ_0 such that $z(\theta) \neq 0$ for all θ, a sequence of agents with the signal H make buying orders until the public belief reaches $\bar{\mu}(H)$.

Phase 2. Euphoria: Subsequent to the Boom phase, a sequence of agents with signals either H or L arrive in the market without making non-zero trading orders.

Phase 3. Trigger: Before μ_t grows bigger than $\bar{\mu}(R)$, an agent with the signal R makes a non-zero selling order.

Phase 4. Panic: Subsequent to the Trigger, agents who have arrived in the market during Euphoria make non-zero trading orders. If they are mostly agents with the signal L making selling orders, a crash occurs.

Now we provide a heuristic argument to prove that the four phases illustrated above constitute the price path consistent with the stylized facts. Since by Lemma 2 any non-zero trading order is fully revealing of the underlying private signal and $p_{t+1} = \mu_t/(\mu_t + (1-\mu_t)\lambda_\theta) > p_t$ for $\theta = H$, the string of agents with the signal H bids up the price in the phase of boom. If enough buying orders are made so that $p_t = \bar{\mu}(H)$, a partial informational cascade including the signals H and L develops strongly indicating the state G. During the euphoria phase, an agent with an L signal is not distinguishable from the one with an H since they both do not place non-zero trading orders. Therefore, the market can accommodate many agents with an L signal while the price still remains high indicating the state G. Moreover, if the true state is B, it is likely that there are more agents with the signal L than H overall, although agents with H signal may have arrived in the market early by chance.

The timing of the trigger of an R signal is important in determining the size of the market crash. If it happens too late, a total informational cascade will develop by Proposition 4. If it arrives too early in the euphoria phase, the price change will be less dramatic since there may not have accumulated enough agents with the signal L. Once the trigger is made, all agents with the signal L make selling orders since their mild skepticism is strengthened by the trigger and an informational avalanche follows, resulting in the panic. The price fall which happens during the panic will be bigger if there are more agents with the signal L than those with H which, in turn, is more likely under the state B. Moreover, the low price will remain so since the best information available in the economy indicates that the state B is more likely to be the true state.

The price volatility during an informational avalanche can be computed as in the following proposition which is assumed to happen subsequent to a partial informational cascade of N periods. The computation confirms that the volatility increases with the length of the informational cascade preceding the informational avalanche.

Proposition 5. *Suppose that a trigger is made in period* $(t-1)$ *after N periods of a partial informational cascade including the signals L and H. Then the variance of the price in period* $(t+1)$ *conditional on state s is computed as*:

$$\mathrm{var}\,[p_{t+1}|s] = \textstyle\sum_{n=0}^{N} \binom{N}{n}\left(\frac{q_{Hs}}{q_{Hs}+q_{Ls}}\right)^{n}\left(\frac{q_{Hs}}{q_{Hs}+q_{Ls}}\right)^{N-n}\left(\frac{p_t}{p_t+(1-p_t)\lambda_n}-p_t\right)^2,$$

where λ_n *is the likelihood of state B against state G conditional on drawing a sample of size N which has n of signal H.*

5.2. *Discussion*

We characterized the market crash as a failure in aggregating dispersed information in the economy. Agents in the model are described as acting rationally based on all information available at the moment of investment decision. Nonetheless, they fail to aggregate their private information correctly because of transaction costs.

The difficulty in deriving the explanation lies in the fact that the small errors each agent may make tend to be cancelled out at the equilibrium. The informational cascade caused by the transaction costs demonstrates how small errors may accumulate systematically instead of cancelling out when agents move sequentially. In contrast, small errors would cancel out if all agents move at once and the market locates the equilibrium price based on the total information. Therefore the model suggests the possibility of a serious flaw in the information aggregation role of a sequential market.

The asset market analysed in the previous section explains the observation that market volatility increases as the trading activity increases. In particular, Jones, Kaul and Lipson (1994) show that the market volatility is correlated with the number of transactions but not as much with the size of each trading order. In our model, the price moves much when trading orders are made by a vast number of agents who have not chosen to trade before due to the transaction cost. Each trading order may not be big since each agent has only one piece of private signal not reflected in the price yet. However the big number of trading orders that occurs in the informational avalanche causes the prices to move dramatically.

In addition, our result suggests a testable hypothesis that after the market has undergone a longer period of a monotone movement, it is more likely to have greater volatility. The magnitude of the price movement in the informational avalanche depends on the number of agents with hidden information as well as the contents of the information. The number of agents with hidden information increases with the length of the informational cascade preceding the unexpected price movement which diffuses the public belief. Therefore there is a bigger probability of drastic price movement if the price has been moving monotonically for a long period.

The model assumes that a potentially large number of trades are executed at a single price in a single period during an informational avalanche and that the transaction cost does not change during such a turbulent time. In reality it is possible that only a part of the trading orders are executed before the price adjusts and that the transaction cost represented by the bid–ask spread gets bigger reflecting higher uncertainty. Since these two possibilities imply that not all traders with hidden information may be able to trade, and consequently they reveal less information, the price movement during the informational avalanche could be smaller in reality than the model predicts.

This model does not derive from the usual story of informed trade and noise trader. Compared to those models, there is less informational asymmetry in the model, yet the market does not correctly aggregate information. In this sense it can be said that this model is based on the minimal amount of informational asymmetry.

The sequence of signals in the characterization may appear very restrictive; it is necessary that agents with favourable signals move first to set an optimistic tone for the market. However, the restrictiveness in the arrival order underscores that the phenomenon would not be observed too often. Moreover the condition characterizing the trade orders in each phase can be relaxed to allow the price to evolve through ups and downs before going up high. It is worthwhile to note that as there are more ups and downs, more information would be revealed so that the uninformative cascades would not occur as easily as otherwise.

The price path from the sequence of private signals as considered in the previous subsection is consistent with the stylized facts which point out the steady rise of price before the crash and the low price after the crash. There is an apparent discrepancy between the price path generated from such a sequence and actual price movements around major crashes, namely the price rise continues until immediately before the crash. However it appears that the present model may have an extended price rise if we allow many signals where moderate signals arrive in the market during the partial informational cascade and at the same time more informative and positive signals bid up the price. Finally a strongly negative signal arrives in the market later to cause a trigger.

The dynamics of market crashes illustrated in the model is standard among real world investors who do not subscribe to the efficient market hypothesis. They usually postulate the transition of market as "Accumulation," "Distribution," and "Liquidation" that correspond roughly to boom, euphoria, and panic in our model. The important difference of our theory is that the same pattern can be generated by rational behaviour of market participants. Hence this paper provides the theoretical framework for the very common belief.

This model is able to explain a variety of features of the 1987 market crash. First the result accords well with the claims of many traders that they were skeptical during the price rise. (Shiller (1988)). Many agents with low signals may have been induced to buy at high prices thus ignoring their private information. This was at least in part due to an uninformative cascade. They regretted their purchase once the state was revealed through the October market crash. Next the fact that the price fall began in the week before the market crash in 1987 can be explained by the model. When the rare signal is revealed, the price begains to fall and subsequently crash occurs only when substantial number of low signals are revealed to the market. Also the modest initial price drop (Fama (1989)) is consistent with the model because the market crash in our model is triggered by one trader whose trading order does not induce a huge price drop by itself.

There have been numerous attempts to explain the international market crash of October 1987. Most notably there have been attempts to find the source of the market crash in the trading strategy. Genotte and Leland (1990) and Jacklin, Kleidon, and Pfleiderer (1990) explain the market crash by asymmetric information about the extent of portfolio insurance. Although they succeed in generating a pattern similar to the market crash, the actual amount of portfolio insurance during the market crash of 1987 seems to suggest otherwise; after the market crash, the surprise was not that there were more portfolio insurers than expected, but that there were fewer insurers. Therefore their explanation is not consistent with the data. Also the crash was observed in countries without portfolio insurance (Roll (1989)). If portfolio insurance was the major reason for the crash in the U.S. we have to find an alternative explanation pertaining to other markets' crashes. In contrast, the present model suggests that crashes may happen in all markets since the sequential trading structure and transaction costs exist in all markets in one form or another.

6. CONCLUSION

This paper explains the market crash by the failure of information aggregation due to the transaction cost in the security market. The result demonstrates that small errors due to small friction may systematically accumulate into a big blunder instead of cancelling each other out along the way. In particular, it shows that a big change in the price can happen without substantial news; although the news that triggers a crash should be relatively substantial, it is not the one that determines the price change subsequent to its revelation. Thus the model predicts that the security price can change dramatically even in the absence of a correspondingly dramatic news.

The model can be regarded as an alternative to the noise trader approach such as Schleifer and Summers (1990). In contrast to the noise trader model in which the behaviour of a certain proportion of agents is not explained endogenously, the model explains all agents' behaviour in a rational fashion. Yet the sequential structure combined with the transaction cost and the dispersed information is shown to generate interesting features in the security price movement not easily explained by the standard rational expectations model.

The model provides a policy implication for designing a more efficient capital market. By executing more orders at once, one can avoid the distortion of the individual investment decision due to the previous uninformative history. In particular, the auction mechanism that is employed when a substantial imbalance exists in the market should be seriously considered because it alleviates the problem from the sequential structure of the market. Paradoxically, a trading mechanism which provides less information about the previous trading orders may help agents fully reveal their private information.

The model leaves a few issues unresolved. Amongst them, the model in principle can generate frenzies as often as crashes although frenzies are avoided by asymmetric design of the model in Section 5. In reality we seldom observe frenzies relative to crashes. It remains to show why frenzies do not happen as often as crashes. A potential answer seems the risk aversion of investors. Under risk aversion it is more difficult to trigger a frenzy than a crash because a surprise of the same degree in the direction of the good state induces a smaller response than the one in the direction of the bad state. Indeed, Chalkley and Lee (1998) show that a model constructed using this intuition is able to produce the desired result.

APPENDIX

Lemma 1. *Suppose $\mu_t = \pi_t^i$. If there is no further transaction cost for trading, then $z_t^i = -x_t^i$.*

Proof. First note that if $\mu_t = \pi_t^i$, then $\mu_{t+s} = \pi_{t+s}^i$ for all $s \geqq 1$ since agents have only one private signal initially and do not acquire new private information subsequently.

Let $t=T+1$. Define

$$\tilde{u}(z\mid\pi,p,W,x)=E\{-\exp[-(W+Y\cdot(x+z)-pz)]\}.$$

The first derivative of $\tilde{u}(z\mid\mu,\pi,W,x)$ is

$$d/dz\,\tilde{u}(z\mid\pi,p,W,x)=E\{\exp[-(W+Y\cdot(x+z)-pz)]\cdot(Y-p)\}.$$

It is easy to check that the second derivative is strictly negative so that the trading order which meets the first order condition is the unique optimum. When $\mu_t=\pi^i_t$ so that $EY-p=0$, the first order condition is met if $x+z=0$. If follows that $z^i_{T+1}=-x^i_{T+1}$ or 0.

Let $t=T$. Taking account of the solution for $t=T+1$, if $z^i_T\neq 0$, the agent maximizes

$$\begin{aligned}&\beta\tilde{u}(z\mid\pi^i_T,p_T,W^i_T,x^i_T)+(1-\beta)EV_{T+1}(\pi^i_{T+1},p_{T+1},W^i_{T+1},x^i_{T+1})\\&\quad=\beta E_T\{-\exp[-(W^i_T+Y\cdot(x^i_T+z^i_T)-p_Tz^i_T)]\}+(1-\beta)\{E_T\{-\exp[-(W^i_T+p_T(x^i_T+z^i_T))]\}\}.\end{aligned}$$

The first derivative is

$$\begin{aligned}&\beta E_T\{-\exp[-(W^i_T+Y\cdot(x^i_T+z^i_T)-p_Tz^i_T)](Y-p_T)\}\\&\quad+(1-\beta)\{E_T\{-\exp[-(W^i_T+p_T(x^i_T+z^i_T)-p_Tz^i_T)](p_{T+1}-p_T)\}\}.\end{aligned}$$

If $x^i_T+z^i_T=0$, the first order condition becomes

$$\exp[-(W^i_T+p_Tx^i_T)]\{\beta E_T(Y-p_T)+(1-\beta)[E_T(p_{T+1}-p_T)]\}=0.$$

since $E_TY-p_T=0$ by hypothesis and $E_T(p_{T+1}-p_T)=E_T(E_{T+1}Y-p_T)=0$ by iterated conditional expectation.

For $t<T$, the proof is identical where we utilize the fact that $E_t(p_{t+s}-p_t)=E_tE_{t+s}Y-p_t=0$ for all $s\geqq 1$ by iterated conditional expectation and by hypothesis. ||

Corollary 1. *Suppose* $\mu_t=\pi^i_t$ *and* $x^i_t=0$. *Then* $z^i_t=0$.

Proof. If $x^i_t=0$, then $z^i_t=0$ from Lemma 1. ||

Lemma 2. *Consider* θ, $\theta'\notin\hat{\Theta}_t$, *such that* $\lambda_\theta\neq\lambda_{\theta'}$ *where* $\hat{\Theta}_t$ *is the informational cascade set in trading round* t. *Then* $z^i_t(\theta)\neq z^i_t(\theta')$ *where* $z^i_t(\theta)$ *is the optimal trading order of agent i with the private signal* θ.

Proof. Let $t=T+1$. If $z=\arg\max_z\tilde{u}(z\mid\mu,\pi,W,x)$, then $dz/d\pi=-(d^2\tilde{u}(z\mid\pi,p,W,x)/dzd\pi/(d^2\tilde{u}(z\mid\pi,p,W,x)/(dz)^2)$. Since the denominator is negative by the second order condition, $dz/d\pi$ has the same sign as the numerator.

$$\begin{aligned}&\frac{d^2\tilde{u}(z\mid\pi,p,W,x)}{dzd\pi}\\&\quad=\frac{d}{d\pi}\{\pi\exp[-(W+(x+z)-pz)]\cdot(1-p)+(1-\pi)\exp[-(W-pz)]\cdot(-p)\}\\&\quad=\exp[-(W+(x+z)-pz)]\cdot(1-p)-\exp[-(W-pz)]\cdot(-p)>0.\end{aligned}$$

Since $d\pi/d\lambda_\theta=-\mu(1-\mu)/(\mu+(1-\mu)\lambda_\theta)^2<0$, $dz/d\lambda_\theta<0$, that is the optimal trading order is strictly decreasing in the likelihood ratio of B against G. Since the agent maximizes $\tilde{u}(z\mid\pi^i_{T+1},p_{T+1},W^i_{T+1},x^i_{T+1})$ in trading round $T+1$, $z^i_{T+1}(\theta)\neq z^i_{T+1}(\theta')$ if $\lambda_\theta\neq\lambda_{\theta'}$.

Suppose that the hypothesis is true for $t+1$. In trading round t, the agent maximizes

$$\begin{aligned}&\beta\tilde{u}(z^i_t\mid\pi^i_t,p_t,W^i_t,x^i_t)+(1-\beta)E_{t+1}V_{t+1}(\pi^i_{t+1},p_{t+1},W^i_{t+1},x^i_{t+1})\\&\quad=\beta E_t\{-\exp[-(W^i_t+Y\cdot(x^i_t+z^i_t)-p_tz^i_t)]\}\\&\qquad+(1-\beta)E_t\{\beta E_{t+1}\{-\exp[-(W^i_t+Y(x^i_t+z^i_t+z^i_{t+1})-p_tz^i_t-p_{t+1}z^i_{t+1})]\}\\&\qquad+(1-\beta)E_t[-\exp[-(W^i_t+p_{t+2}(x^i_t+z^i_t+z^i_{t+1})-p_tz^i_t-p_{t+1}z^i_{t+1})]]\},\end{aligned}$$

where we utilize the fact that non-zero trading order in trading round $t+1$ is fully revealing by hypothesis so that Lemma 1 implies that there is no further trading after $t+2$.

For the same reason as the case for $t=T+1$, it suffices to check the sign of the cross derivative of the objective function with respect to z and π. The term on the second line of the objective function has the same form as $\tilde{u}(z|\pi, p, W, x)$ except the expectation should be taken with respect to π^i_{t+1}. Since

$$\frac{d\pi^i_{t+1}}{d\pi^i_t}=\frac{\lambda_{p_{t+1}}}{(\pi^i_t+(1-\pi^i_t)\lambda_{p_{t+1}})^2}>0,$$

where $\lambda_{p_{T+1}}$ is the likelihood of B against G conditional on p_{T+1}, the cross derivative is positive as was shown for $u(z|\pi, p, W, x)$.

Finally the cross derivative of the last line can be further manipulated to:

$$(d/d\pi^i_t)\{E_t[\exp[-(W^i_t+p_{t+2}(x^i_t+z^i_t+z^i_{t+1})-p_tz^i_t-p_{t+1}z^i_{t+1})](p_{t+2}-p_t)]\}$$
$$=E_t[\exp[-(W^i_t+p_{t+2}(x^i_t+z^i_t+z^i_{t+1})-p_tz^i_t-p_{t+1}z^i_{t+1})]dp_{t+2}/d\pi^i_t,$$

by the Envelope Theorem. Since $dp_{t+2}/d\pi^i_t \geqq 0$, the cross derivative in the last line is also positive. The term following β in the derivative is positive as shown for the case $t=T+1$. Therefore $dz^i_t/d\pi^i_t>0$ and thus $z^i_t(\theta)\neq z^i_t(\theta')$ if $\lambda_\theta \neq \lambda_{\theta'}$ and the proof is complete. ||

Proposition 1. $E[p_{t+1}|p_t]=p_t$.

Proof. The proof follows from the observation that the price process is a Doob process (Karlin and Taylor (1975)) since it is the sequence of the expectation of a random variable (the liquidation value) conditional on a sequence of growing information sets. ||

Proposition 2. *There exist unique* $1>\bar{\mu}(\theta)>0$ *and* $1>\underline{\mu}(\theta)>0$ *such that for* $\bar{\mu}(\theta)\geqq\mu\geqq\underline{\mu}(\theta)$ *the agent newly arriving in trading round t with a private signal* θ *places a non-zero trading order.*

Proof. Fix a private signal θ. Notice that $p_t=\mu_t\cdot 1+(1-\mu_t)\cdot 0=\mu_t$. When $\mu_t=0$ or 1, $\pi^i_t=\mu_t$ for any signal θ and hence $\Phi(\pi^i_t,p_t)=-1-c<-1$ where the agent can guarantee the expected payoff of -1 by never making a non-zero trading. It follows that when $\mu_t=0$ or 1, the agent chooses not to trade. Using the Envelope Theorem it is easy to see that $\Phi(\pi^i_t,p_t)$ is concave in p_t. Therefore there exist $\bar{\mu}(\theta)$ close to 1 and $\underline{\mu}(\theta)$ close to 0 such that if either $1\geqq\mu\geqq\bar{\mu}(\theta)$ or $0\leqq\mu\leqq\underline{\mu}(\theta)$, $\Phi(\pi^i_t,p_t)\leq -1$ so that $z^i_t(\theta)=0$. Moreover $\bar{\mu}(\theta)$ and $\underline{\mu}(\theta)$ are unique because $\Phi(\pi^i_t,p_t)$ is unique. ||

Proposition 3. *Suppose that in trading round t,* $z^i_t\neq 0$ *for some* $i<t$. *Then*;

1. $\mathrm{Var}[p_{t+1}]\gneqq\mathrm{Var}[E[Y|p_t,z^t_t]]$;
2. $E[(p_{t+1}-Y)^2]\leqq E[(E[Y|p_t,z^t_t]-Y)^2]$ *where Y is the true liquidation value.*

Proof. 1. Denote the non-zero trading orders placed by agents who were already in the market by $\tilde{z}_t$, that is, $z_t=\{z^i_t|z^i_t\neq 0$ for some $i<t\}$. We write $\mathrm{Var}[p_{t+1}]=E[E[Y|p_t,z_t,z^t_t]-p_t]^2$ and $\mathrm{Var}[E[Y|p_t,z^t_t]]=E[E[Y|p_t,z^t_t]-p_t]^2$ due to the Martingale property of the price sequence. Also the law of iterated conditional expectations implies that $E[E[Y|p_t,z_t,z^t_t]|p_t,z^t_t]=E[Y|p_t,z^t_t]$. If follows that

$$\begin{aligned}\mathrm{Var}[p_{t+1}]&=E[E[Y|p_t,\tilde{z}_t,z^t_t]-p_t]^2\\ &\geqq E[E[Y|p_t,z^t_t]-p_t]^2\\ &=\mathrm{Var}[E[Y|p_t,z^t_t]].\end{aligned}$$

where the inequality follows from Jensen's inequality.

2. Now consider the problem of minimizing the mean squared error from predicting the liquidation value conditional on p_t,z_t,z^t_t where z_t denotes the non-zero trading order of the agent $i, i<t$: $\min E[(x-Y|p_t,z_t,z^t_t]$.

The solution to the problem is the expectation conditional on all available information, that is, $x=E[Y|p_t,z_t,z^t_t]=p_{t+1}$. The mean squared error for such an optimal statistic should be smaller than the expectation conditional only on p_t,z^t_t. Therefore $E[(p_{t+1}-Y)^2]\leqq E[(E[Y|p_t,z^t_t]-Y)^2]$. ||

Proposition 4. 1. $\Pr\{\lim_{t\to\infty}z^t_t(\theta)=0$, *for all* $\theta\in\Theta\}=1$.
2. $\Pr\{\lim_{t\to\infty}\arg\min_Y|Y-p_t|\neq Y\}>0$ *where Y is the true liquidation value.*

Proof. 1. Notice that the event $\{\lim_{T\to\infty}z^T_T(\theta)=0$, for all $\theta\in\Theta\}$ is equivalent to the event that the price converges to a constant. In the remaining we show that the price indeed converges to a constant with probability 1.

The price sequence converges to a limit by the Martingale Convergence Theorem since it is a Martingale process. To show that it converges to a constant, observe that the price process has 2 absorbing states, one that is close enough to 1 and the other to 0 that the inequality (4) holds true for no $\theta \in \Theta$. Therefore the price sequence converges to either of absorbing states which is constant.

2. The proof is an adaptation of the theorem that a total informational cascade always has a strictly positive probability of being non-fully revealing. (Lee (1993*a*)). ||

Proposition 5. *Suppose that a trigger is made in period* $(t-1)$ *after N periods of a partial informational cascade including the signals L and H. Then the price variance conditional on state s is computed as*

$$\operatorname{var}[p_{t+1}|s] = \textstyle\sum_{n=0}^{N} \binom{N}{n} \left(\dfrac{q_{Hs}}{q_{Hs}+q_{Ls}}\right)^{n} \left(\dfrac{q_{Hs}}{q_{Hs}+q_{Ls}}\right)^{N-n} \left(\dfrac{p_t}{p_t+(1-p_t)\lambda_n} - p_t\right)^{2},$$

where λ_n *is the likelihood of state B against state G conditional on drawing n of signal H out of total sample of N.*

Proof. First notice that the distribution of N signal draws during the partial informational cascade follows a binomial distribution whose probability of success is $q_{Hs}/(q_{Hs}+q_{Ls})$. By Proposition 1, $E[p_{t+1}|p_t]=p_t$. Hence the variance of the price in period $t+1$ is computed as $(p_{t+1}-Ep_{t+1})^2$ $(=(p_t/(p_t+(1-p_t)\lambda_n)-p_t)^2)$ multiplied by the binomial distribution of the parameter $q_{Hs}/(q_{Hs}+q_{Ls})$. ||

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REFERENCES

AVERY, C. and ZEMSKY, P. (1995), "Multi-Dimensional Uncertainty and Herd Behavior in Financial Markets" (*Mimeo*).

BANERJEE, A. (1992), "A Simple Model of Herd Behavior", *Quarterly Journal of Economics*, **107**, 797–817.

BIKHCHANDANI, S., HIRSHLEIFER, D. and WELCH, I. (1992), "A Theory of Fads, Fashions, Customs and Cultural Change as Informational Cascades", *Journal of Political Economy*, **100**, 992–1026.

BULOW, J. and KLEMPERER, P. (1994), "Rational Frenzies and Crashes", *Journal of Political Economy*, **102**, 1–23.

CAPLIN, A. and LEAHY, J. (1991), "Business as Usual, Market Crashes and Wisdom After the Fact", *American Economic Review*.

CHALKLEY, M. and LEE, I. H. (1988), "Learning and Asymmetric Business Cycles", *Review of Economic Dynamics*, **1**, 623–645.

DEVENOW, A. and WELCH, I. (1995), "Rational Herding in Financial Economics" (IFA Working Paper 218–1995, LBS).

FAMA, E. (1989), "Perspective on October 1987, or What Did We Learn from the Crash?", in *Black Monday and the Future of Financial Markets*, R. W. Kamphuis, Jr., R. C. Kormendi and J. W. Henry Watson (eds.) (Chicago: The Mid-America Institute for Public Policy Research).

FRENCH, K. R. (1988), "Crash-Testing the Efficient Market Hypothesis", *NBER Macroeconomic Annual*, 277–285.

FROOT, K. A., SCHARFSTEIN, D. and STEIN, J. (1990), "Herd on the Street: Informational Inefficiencies in a Market with Short-Term Speculation" (NBER Working Paper No. 3250).

GARBER, P. M. (1989), "Tulipmania", *Journal of Political Economy*, **97**, 535–560.

GENOTTE, G. and LELAND, H. (1990), "Market Liquidity, Hedging, and Crashes", *American Economic Review*, **80**, 999–1021.

GROSSMAN, S. J. (1988), "Analysis of the Implications for Stock and Future Price Volatility of Program Trading and Dynamic Trading Strategies", *Journal of Business*, **61**, 275–298.

GUL, F. and LUNDHOLM, R. (1995), "Endogenous Timing and the Clustering of Agent's Decisions", *Journal of Political Economy*, **103**, 1039–1066.

JACKLIN, C. J., KLEIDON, A. W. and PFLEIDERER, P. (1992), "Underestimation of Portfolio Insurance and the Crash of October 1987", *Review of Financial Studies*, **5**, 35–63.

JONES, C., KAUL, G. and LIPSON, M. (1994), "Transaction, Volume, and Volatility", *Review of Financial Studies*, **7**, 631–651.

KARLIN, S. and TAYLOR, H. M. (1975) *A First Course in Stochastic Processes*, 2nd ed. (New York: Academic Press).

KINDLEBERGER, C. (1989) "*Manias*, *Panics*, *and Crashes*, (New York: Basic Books).

KYLE, A. S. (1985), "Continuous Auctions and Insider Trading", *Econometrica*, **53**, 1315–1335.

LEE, I. H. (1993*a*), "On the Convergence of Informational Cascades", *Journal of Economic Theory*, **61**, 395–411.

LEE, I. H. (1993*b*), "Market Crashes and Informational Avalanches" (Discussion Paper 9312, University of Southampton).

MILLER, M. H, (1991) *Financial Innovations and Market Volatility* (Oxford: Basil Blackwell).

ROLL, R. W. (1989), "The International Crash of October 1987", in *Black Monday and the Future of Financial Markets*, R. W. Kamphuis, Jr., R. C. Kormendi and J. W. Henry Watson (eds.), (Chicago: The Mid-America Institute for Public Policy Research).

ROMER, D. (1993), "Rational Asset-Price Movements Without News", *American Economic Review*, **83**, 1112–1130.

SCHLEIFER, A. and SUMMERS, L. H. (1990), "The Noise Trader Approach to Finance", *Journal of Economic Perspectives*, **4**, 19–33.

SHILLER, R. J. (1988), "Portfolio Insurance and Other Investor Fashions as Factors in the 1987 Stock Market Crash", *NBER Macroeconomic Annual*, 287–295.

SMITH, L. and SORENSEN, P. (1996), "Pathological Outcomes of Observational Learning" (Mimeo, MIT).

ZEIRA, J. (1993), "Informational Overshooting, Boom, and Crashes" (Mimeo, The Hebrew University of Jerusalem).